MAT 303 - Calculus IV with Applications

Practice Midterm II

Stony Brook University
Fall 2023

[15 pts] **Problem 1.**

[5 pts] (i) Find the general solution of the differential equation

$$y^{(3)} - 2y'' + y' = 0.$$

[5 pts] (ii) Solve the initial value problem

$$y^{(3)} - 2y'' + y' = 0; \quad y(0) = 3,\ y'(0) = 4,\ y''(0) = 5.$$

[5 pts] (iii) Find the general solution of the differential equation

$$y^{(3)} - 2y'' + y' = 1 + e^x.$$

[20 pts] **Problem 2.**

[5 pts] (i) Consider a spring-mass-dashpot system with spring constant $k = 50$, mass $m = 2$, and damping constant $c = 12$. Write a differential equation for the position function $x(t)$ of the mass and determine whether the motion is underdamped, critically damped, or overdamped.

[5 pts] (ii) Suppose that at time $t = 0$ the mass has displacement $x_0 = 1$ and we push it to the right with a speed of 1. Find the formula for the position function. Then explain the characteristics of the motion; what is the (variable) amplitude of the motion, if any, the (pseudo)frequency, if any, and the phase angle, if any?

[5 pts] (iii) Suppose that we remove the dashpot and an external force of the form $F(t) = 2\cos\omega t$ acts on the mass, where ω is a constant. For which value of ω do we have resonance? For that value of ω find the formula of the displacement $x(t)$ (give a general solution).

[5 pts] (iv) If we put the dashpot back in place while keeping an external force of the form $F(t) = 2\cos\omega t$, you are given that the steady periodic solution is

$$x_{sp}(t) = \frac{1}{\sqrt{(25-\omega^2)^2 + 36\omega^2}}\cos(\omega t - \alpha).$$

Explain why in that case we do not have resonance for any value of ω. You are given that the smallest possible value of $(25-\omega^2)^2 + 36\omega^2$ occurs when $\omega = \sqrt{7}$. What phenomenon occurs then?

[15 pts] **Problem 3.**

[5 pts] (i) You are given that $y_1 = x$ is a solution of the homogeneous differential equation

$$y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0,$$

where $x > 0$. Let $y_2 = xv(x)$ for some function v. Specify a function v with $v' \neq 0$ so that y_2 is a solution to the differential equation.

[5 pts] (ii) Compute the Wronskian of the functions y_1 and y_2 and show that they are linearly independent.

[5 pts] (iii) Use the method of variation of parameters to find a particular solution of the differential equation

$$y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 72x^3.$$